SN – 353

V Semester B.A./B.Sc. Examination, November/December 2017
(Semester Scheme) (CBCS) (2016 – 17 & Onwards)
(Fresh + Repeaters)
MATHEMATICS – V

Time : 3 Hours Max. Marks : 70

***Instruction** : Answer **all** questions.*

PART – A

Answer **any five** questions : **(5×2=10)**

1 a) In a ring $(R, +, \cdot)$ prove that $\forall a, b, c \in R$, $a \cdot (b - c) = a \cdot b - a \cdot c$.

b) Show that the set of even integers is not an ideal of the ring of rational numbers.

c) Prove that every field is a principal ideal ring.

d) If $\vec{F} = yz\hat{i} + zx\hat{j} + xy\hat{k}$, show that $\vec{F}$ is irrotational.

e) Find the maximum directional derivative of $x\sin z - y\cos z$ at (0, 0, 0).

f) Prove that $E\nabla = \nabla E = \Delta$.

g) Construct the Newton's divided difference table for the following data :

x	4	7	9	12
f(x)	– 43	83	327	1053

h) Using Trapezoidal rule to evaluate $\int_0^1 \frac{dx}{1+x}$ where

x	0	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	1
y = f(x)	1	0.8571	0.75	0.6667	0.6	0.5455	0.5

P.T.O.

PART – B

Answer **two full** questions : **(2×10=20)**

2. a) Prove that the set $R = \{0, 1, 2, 3, 4, 5\}$ is a commutative ring with respect to '$\oplus_6$' and '$\otimes_6$' as the two compositions.

   b) Prove that a ring R is without zero divisors if and only if the cancellation laws hold in R

OR

3. a) Show that an ideal S of the ring of integers $(z, +, \cdot)$ is maximal if and only if S is generated by some prime integer.

   b) Prove that a commutative ring with unity is a field if it has no proper ideals.

4. a) If R is a ring and $a \in R$, let $I = \{x \in R / ax = 0\}$ prove that I is a right ideal of R.

   b) If $f : R \to R'$ be a homomorphism with kernel K, then prove that f is one-one if and only if $K = \{0\}$.

OR

5. a) Let $R = R' = C$ be the field of complex numbers. Let $f : R \to R'$ be defined by $f(z) = \bar{z}$ where $\bar{z}$ is the complex conjugate of z, show that f is an isomorphism.

   b) Prove that every homomorphic image of a ring R is isomorphic to some residue class (quotient) ring thereof.

PART – C

Answer **two full** questions : **(2×10=20)**

6. a) Prove that $\nabla^2(f(r)) = f''(r) + \frac{2}{r} f'(r)$, where $r^2 = x^2 + y^2 + z^2$.

   b) Find the unit normal to the surface $x^3 + y^3 + 3xyz = 3$ at the point $(1, 2, -1)$.

OR

7. a) Show that Curl $\left[\vec{r}\times(\vec{a}\times\vec{r})\right] = 3\,\vec{r}\times\vec{a}$ where $\vec{a}$ is constant vector and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

b) If the vector $\vec{F} = (3x + 3y + 4z)\,\hat{i} + (x - ay + 3z)\,\hat{j} + (3x + 2y - z)\,\hat{k}$ is solenoidal, find 'a'.

8. a) Prove that $\nabla^2\left(\frac{1}{r}\right) = 0$, where $r^2 = x^2 + y^2 + z^2$.

b) If $\vec{F} = \nabla(2x^3 y^2 z^4)$, find Curl $\vec{F}$ and hence verify that Curl $(\nabla\phi) = 0$.

OR

9. a) If ϕ is a scalar point function and $\vec{F}$ is a vector point function, prove that

$$\text{div}(\phi\vec{F}) = \phi\,\text{div}\,\vec{F} + \text{grad}\,\phi\cdot\vec{F}$$

b) Find Curl (Curl $\vec{F}$) if $\vec{F} = x^2 y\,\hat{i} - 2xz\,\hat{j} + 2yz\,\hat{k}$.

PART – D

Answer **two full** questions : (2×10=20)

10. a) Use the method of separation of symbols to prove that

$$u_0 + u_1 x + u_2 x^2 + \ldots \text{ to } \infty$$

$$= \frac{u_0}{1-x} + \frac{x\Delta u_0}{(1-x)^2} + \frac{x^2\Delta^2 u_0}{(1-x)^3} + \ldots \text{ to } \infty.$$

b) i) Evaluate $\Delta^{10}\,[(1 - ax)(1 - bx^2)(1 - cx^3)(1 - dx^4)]$.

ii) Express $f(x) = 3x^3 + x^2 + x + 1$ as a factorial polynomial (taking $h = 1$).

OR

11. a) Find a second degree polynomial which takes the following data :

x	1	2	3	4
f(x)	–1	–1	1	5

b) Find f(1.9) from the following table :

x	1	1.4	1.8	2.2
f(x)	2.49	4.82	5.96	6.5

12. a) Using Lagrange's interpolation formula find f(6) for the following data :

x	2	5	7	10	12
f(x)	18	180	448	1210	2028

b) Using Simpson's $\frac{3}{8}^{th}$ rule evaluate $\int_0^{0.6} e^{-x^2}\,dx$ by taking 6 sub intervals.

OR

13. a) Following is the table of the normal weights of babies during the first few months of life.

Age in months	2	5	8	10	12
Weight in kgs	4.4	6.2	6.7	7.5	8.7

Estimate the weight of a baby of 7 months old using Newton's divided difference table.

b) Obtain an approximate value of $\int_0^6 \frac{dx}{1+x^2}$ by Simpson's $\frac{1}{3}^{rd}$ rule.

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